

Application of Local Non-stationary LMC for Gradational Boundaries

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Abstract

Gradational or soft boundaries are well spread in several types of geological settings due to the transitional nature of geological mineralisation processes. Contacts where grades change transitionally across the boundary are usually characterized by a non-stationary behavior of the variable of interest in the proximities of the boundary, that is, the mean, variance or covariance are no longer constant within a zone of influence of one rock type into the other, and their values depend on the location relative to the boundary.

Conventional estimation usually treats soft boundaries between geological units as hard boundaries, primarily due to the limitations of current estimation and simulation procedures. A new technique has been proposed to account for non-stationary boundaries between multiple rock types in kriging estimation. Non-stationary features in the vicinity of a boundary are parameterized into a local model of coregionalization. With a legitimate spatial model, estimation of grades can be performed using a form of non-stationary cokriging. This contribution reviews the application and implementation of this new technique to a real deposit.

Background

Geostatistical techniques, such as kriging and simulation, make strong assumptions of stationarity in the mean and the variance over the domain of interest. Unfortunately, geological nature usually does not reflect this assumption and we are forced to subdivide our model area into stationary regions that have some common geological controls and similar statistical properties.

Some deposits will have a very simple geometry and mineralisation pattern, making the choice of estimation domains straightforward from geological units. But in general, mineralisation is not entirely defined by a single geological control; structural and/or lithological influence on grade distribution and multiple events of mineralisation produce some degree of overlapping between geological units. Complex mineralisation patterns usually translate to gradational boundaries by a non-stationary behavior of the mean, variance or covariance.

Copper mineralisation in porphyry-type deposits occurs in general as disseminated and stockwork mineralisation. Copper grades usually change transitionally across the geological boundaries. This type of deposit is usually embedded in a large-scale fault system that gives a structural signature to contacts between domains. Since faults are usually active through all the deposit formation, some mineralisation occurs along extension fractures around the major faults, leading to a non-stationary behavior of grades towards a boundary. Post mineralisation processes like

leaching for example, which occurs preferentially along faults as well; also result in a non-stationary behavior in the vicinity of contacts.

The correct representation of soft boundaries should ensure the reproduction of the correlation of the grades across the boundary and ensure reproduction of non-stationary variations of the mean, variance and covariance in the zone of influence of each rock type. Non-stationary cokriging in the presence of geological boundaries accounts for stationary variables within rock types and additional non-stationary factors near boundaries. The technique involves the following distinct phases: (i) identification of the rock types and boundary zones based on geological modeling and the timing of different geological events, (ii) optimization of the non-stationary components of mean and variance in the boundary zone given the stationary statistical parameters of each domain and the data in the boundary region, (iii) decomposition of the covariance model into stationary and non-stationary components of a linear model of coregionalization and optimization of the latest, and (iv) estimation of grades using non-stationary cokriging. This methodology provides an appealing alternative when complex contacts between different rock types exist.

Application

To show the application and the steps involved in the non-stationary cokriging in the presence of soft boundaries for a real deposit. A 3-D example was built using the geological model of a porphyry copper deposit from Northern Chile (Figure 1), but grades were simulated.

Data generation

An unconditional simulation was generated and transformed to a non-standard normal distribution for each rock type.

$$\begin{aligned}
 Y_2 &\sim N(3.0, 1.2) \text{ with } \gamma_{Y_2}(\mathbf{h}) = 0.2 + 0.8 \cdot Sph_{\begin{pmatrix} h_{\max}=200 \\ h_{\min}=200 \\ vert=100 \end{pmatrix}}(\mathbf{h}) \\
 Y_3 &\sim N(1.0, 1.35) \quad \text{with } \gamma_{Y_3}(\mathbf{h}) = 0.1 + 0.9 \cdot Sph_{\begin{pmatrix} h_{\max}=400 \\ h_{\min}=400 \\ vert=100 \end{pmatrix}}(\mathbf{h}) \\
 Y_4 &\sim N(1.5, 0.5) \text{ with } \gamma_{Y_4}(\mathbf{h}) = 0.1 + 0.9 \cdot Sph_{\begin{pmatrix} h_{\max}=500 \\ h_{\min}=500 \\ vert=100 \end{pmatrix}}(\mathbf{h}) \\
 Y_5 &\sim N(0.25, 0.05) \quad \text{with } \gamma_{Y_5}(\mathbf{h}) = 0.2 + 0.8 \cdot Sph_{\begin{pmatrix} h_{\max}=250 \\ h_{\min}=250 \\ vert=100 \end{pmatrix}}(\mathbf{h})
 \end{aligned}$$

All variograms are rotated in 90° azimuth and dip of 35° following the principal anisotropy in the deposit. Rock type 1 corresponds to a leached zone, rock type 2 to a zone of secondary enrichment, rock type 3 and 4 to units of primary mineralisation corresponding to two intrusive events, and rock type 5 to a peripheral primary mineralisation. The values of rock type 1 were set to a default value (-9) since this unit has no economical interest.

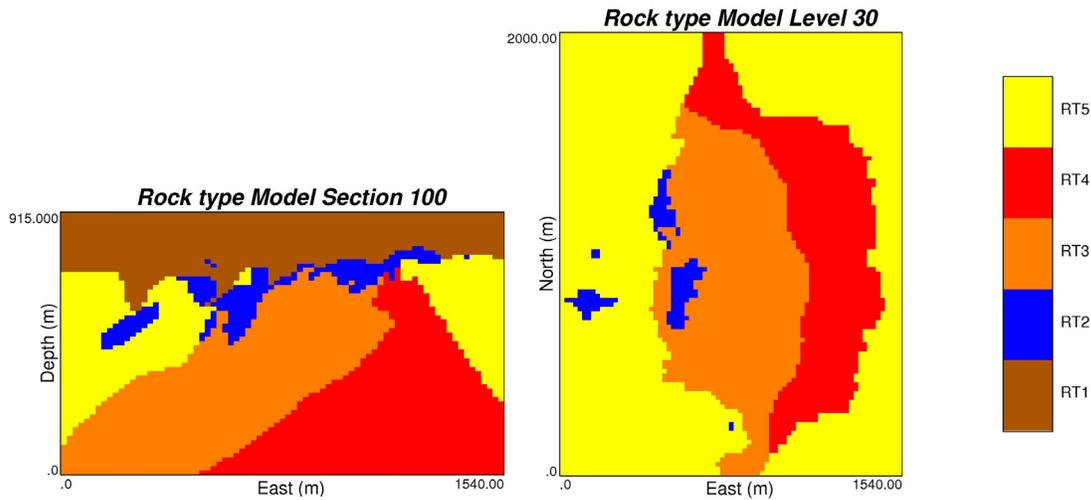


Figure 1: Categorical rock type model of a porphyry copper deposit in Northern Chile.

Non-stationary cokriging requires a rock type model with the boundary zone and distance to boundary assigned to each block. For this the FORTRAN program `boundmod` was used (details of this and the other programs are given in the appendix). The program requires the matrix of maximum distance of influence and a set of precedence rules as input parameters. Considering the geology of the deposit the matrix of maximum distance of influence (in meters) between rock types was chosen as follows:

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 30 & 30 & 30 \\ 0 & 0 & -1 & 120 & 60 \\ 0 & 0 & 90 & -1 & 90 \\ 0 & 0 & 30 & 30 & -1 \end{bmatrix}$$

The boundary zones defined by the contact between the primary mineralisation units was assumed to be more extended than the ones defined by the secondary mineralisation, since they correspond to a wide fault zone. Also the maximum distance of influence of zero of rock type 3 to 5 into rock type 2 reflects that the secondary mineralisation grades into the primary mineralisation but not the opposite.

The set of precedence rules also reflects the timing of the mineralisation; the influence of rock type 2 is the youngest:

$$\begin{array}{l}
4-5 \\
3-4 \\
4-3
\end{array}
\left. \vphantom{\begin{array}{l} 4-5 \\ 3-4 \\ 4-3 \end{array}} \right\} \text{primary mineralization}$$

$$\begin{array}{l}
5-2 \\
4-2 \\
3-2
\end{array}
\left. \vphantom{\begin{array}{l} 5-2 \\ 4-2 \\ 3-2 \end{array}} \right\} \text{secondary mineralization}$$

For the same section and level shown in Figure 1 the boundary zones and distance to boundary are shown in Figure 2.

An additional unconditional simulation was generated to build the non-stationary zones around the boundaries:

$$Y_6 \sim N(0,1) \quad \text{with } \gamma_{Y_6}(\mathbf{h}) = 0.1 + 0.9 \cdot Sph_{\left(\begin{array}{l} h \max=400 \\ h \min=400 \\ vert=50 \end{array} \right)}(\mathbf{h})$$

The variogram of this variable is also rotated in 90° azimuth and dip of 35°.

Y_6 was transformed to a non-stationary variable (Figure 3) using the following expression:

$$Y_6^*(\mathbf{u}_i) = \left[\frac{(dmax_{kp} - d(\mathbf{u}_i))}{dmax_{kp}} \cdot b_{kp} \right] \cdot Y_6(\mathbf{u}_i) + \left[\frac{(dmax_{kp} - d(\mathbf{u}_i))}{dmax_{kp}} \cdot a_{kp} \right]$$

where the distance to the boundary as well as the boundary zone were obtained from the rock type model described above. For each boundary zone, a mean and variance intercept were chosen trying to reproduce the real trends in the deposit:

Boundary Zone (RT-BRT)	Intercept a_{kp}	Intercept b_{kp}
3-2	5.0	2.0
4-2	5.0	2.0
5-2	5.0	2.0
3-4 (and 4-3)	3.0	1.0
3-5 (and 5-3)	0.5	0.5
4-5 (and 5-4)	1.0	1.0

Outside the boundary zone this variable was set to zero, that way, it can be added to the grid file with the simulated values Y_2 to Y_5 to obtain the reference (Figure 4).

Since variables Y_2 to Y_5 were transformed to non-standard normal distributions some negative values occurred. All negative grades (4.3%) were set to zero. Finally the reference grid file was sampled on a 100x100x1 grid to obtain the conditioning data for kriging.

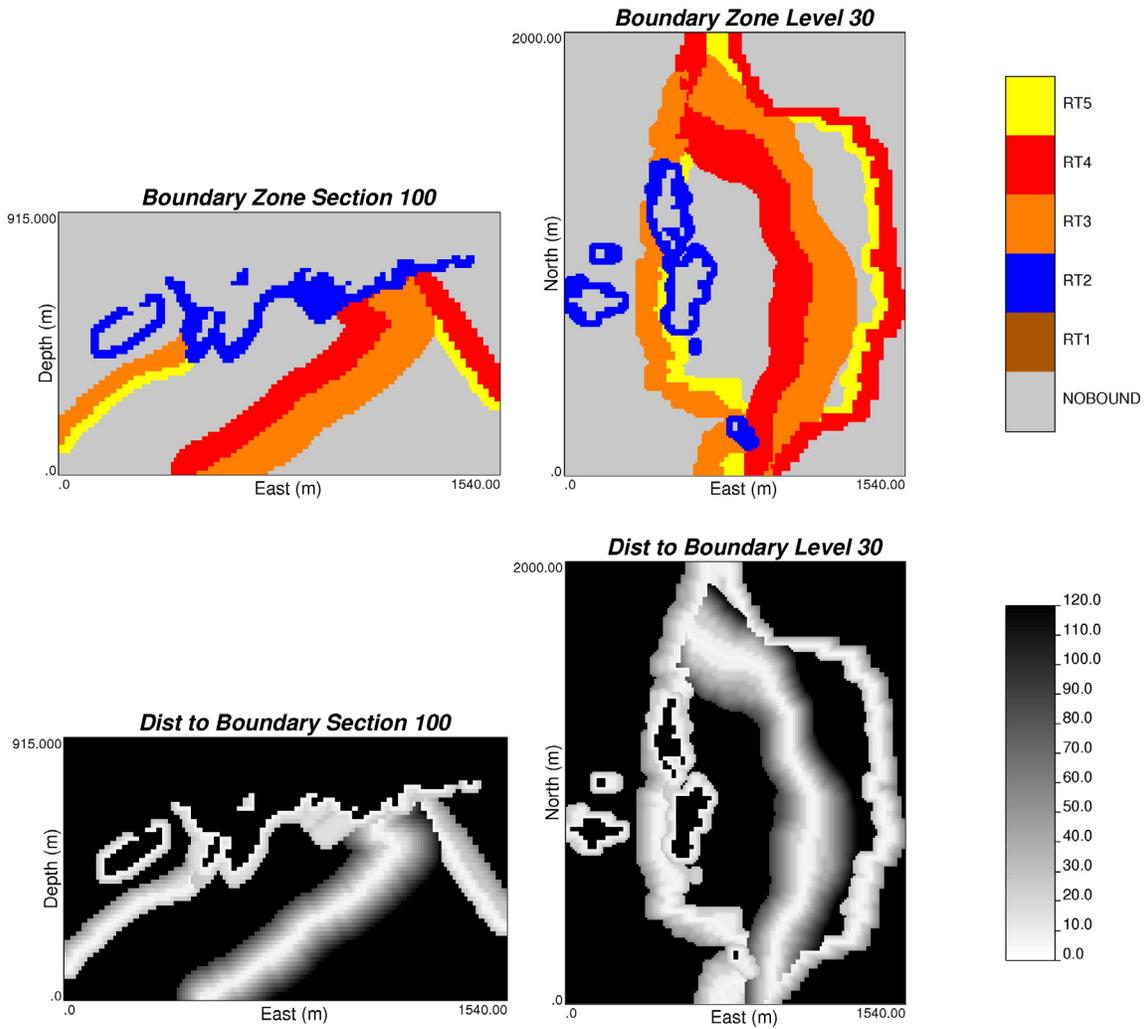


Figure 2: Boundary zone (up) and distance to boundary (bottom) maps for the same section and level of the rock type model of Figure 5.1. The default code (NO BOUND) is given to locations beyond the corresponding maximum distance of influence. Distances beyond the maximum distance of influence are set to the default value 9999, in black.

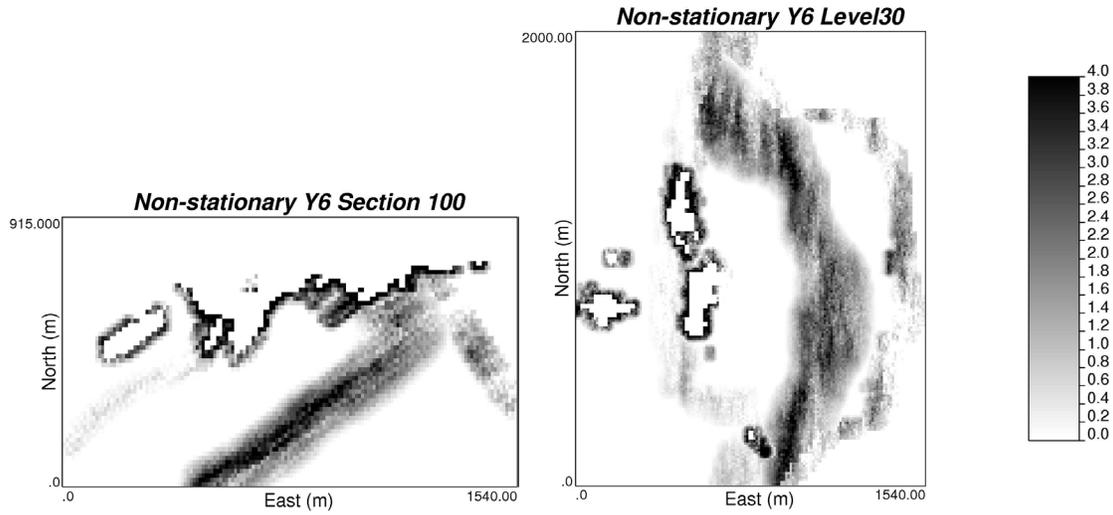


Figure 3: Transformed variable $Y_6(\mathbf{u})$, blocks outside a boundary zone were set to zero.

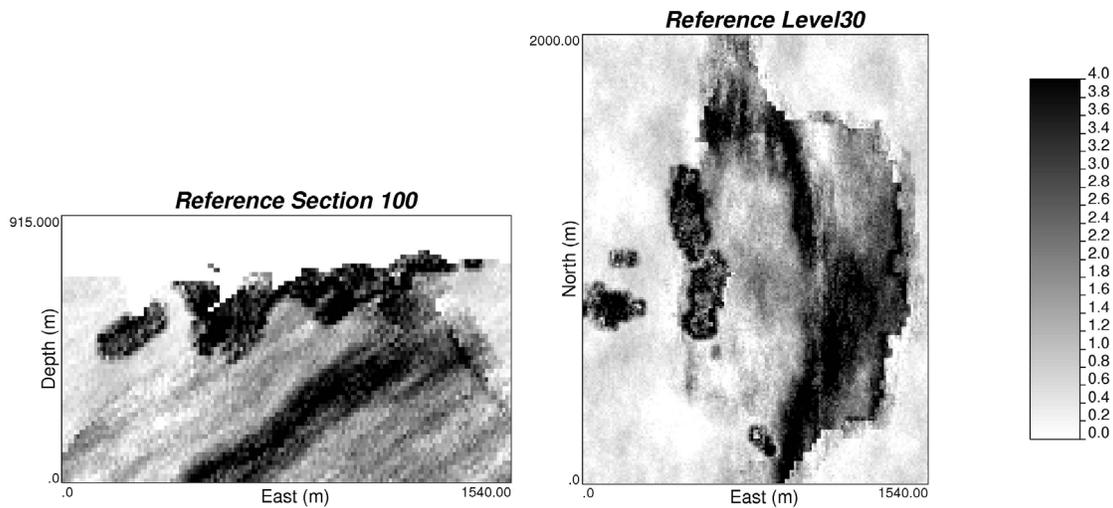


Figure 4: Section and Level maps of the reference distribution. Values from rock type 1 were assigned a default value of -9 since this unit is of no economic interest.

Estimation

Prior to kriging, we need the optimum mean, variance and covariance range for the non-stationary boundaries from the conditioning data. The `opt_mean` (appendix) output (Figure 5) shows that the stationary means of each rock type and the a intercepts are well reproduced for each boundary zone compared with the reference. The b intercepts corresponding to the non-stationary variance are acceptably well reproduced for all boundaries; Figure 6 shows the output of `opt_var` program (appendix). Mismatch with target parameters increase for the non-stationary variance intercepts and then for the covariance range, since at each

optimization step the parameter became more sensitive to statistical fluctuations in the previously optimized parameters.

```

Optimized Means
4
RT
Boundary_RT
Stationary_Mean
NonStationary_Intercept
1 2 -9.0000 0.0000
1 3 -9.0000 0.0000
1 4 -9.0000 0.0000
1 5 -9.0000 0.0000
2 1 3.1180 0.0000
2 3 3.1180 5.0883
2 4 3.1180 5.7429
2 5 3.1180 4.9228
3 1 0.91914 0.0000
3 2 0.91914 5.0883
3 4 0.91914 2.9431
3 5 0.91914 0.60057
4 1 1.6637 0.0000
4 2 1.6637 5.7429
4 3 1.6637 2.9431
4 5 1.6637 1.1153
5 1 0.27290 0.0000
5 2 0.27290 4.9228
5 3 0.27290 0.60057
5 4 0.27290 1.1153

```

Figure 5: Output file of opt_mean program.

In order to find the optimum range of the non-stationary component for each boundary zone, we need: the variogram models of the stationary portions of each rock type and the shape and nugget effect of the relative standardized variogram. The relative standardized variogram corresponds to the stationary shape that will be scaled by the non-stationary standard deviation at each location.

```

Optimized Variances
4
RT
Boundary_RT
Stationary_Variance
NonStationary_Intercept
1 2 0.0000 0.0000
1 3 0.0000 0.0000
1 4 0.0000 0.0000
1 5 0.0000 0.0000
2 1 0.99811 0.0000
2 3 0.99811 3.0120
2 4 0.99811 2.3062
2 5 0.99811 2.5425
3 1 0.27125 0.0000
3 2 0.27125 3.0120
3 4 0.27125 0.87515
3 5 0.27125 0.18306
4 1 0.52029 0.0000
4 2 0.52029 2.3062
4 3 0.52029 0.87515
4 5 0.52029 0.56958
5 1 0.43380E-01 0.0000
5 2 0.43380E-01 2.5425
5 3 0.43380E-01 0.18306
5 4 0.43380E-01 0.56958

```

Figure 6: Output file of opt_var program.

The samples that belong to the stationary portion of each rock type were selected to calculate and model the variograms in the three principal directions. The stationary variance of each rock type was set as the sill. The models are close to the original variograms used in the unconditional simulations except for the minimum horizontal range of rock type 4 that was calculated quite high (Table 1).

Rock type	Structure Type	Sill Contribution	Range Hmax	Range Hmin	Range Vert
2	Nugget Effect	0.2503			
	Spherical	0.7477	250.0	250.0	119.0
3	Nugget Effect	0.0300			
	Spherical	0.2400	531.0	373.0	200.0
4	Nugget Effect	0.0476			
	Spherical	0.4724	498.0	627.0	152.0
5	Nugget Effect	0.0092			
	Spherical	0.0308	267.0	248.0	136.0

Table 1: Variogram models for the stationary regions of rock type 2 to 5.

The shape, anisotropies and nugget effect of the relative standardized variogram of the non-stationary boundaries were chosen as the same as the variogram used to generate the Y_6 variable. Normally the modeler will have to pick these values based on the geology of the deposit. If that were the case here, the parameters would be similar to the chosen ones, the non-stationary zone are strongly control by the fault system that coincides with the major anisotropy, most of the spatial correlation is explained by spherical models, only the nugget effect is difficult to assess.

The optimization of the covariance ranges gives reasonable results (Figure 7) for all boundary zones except for the boundary between rock type 2 and rock type 4, which seems a little too high. This result is likely due to the influence of the minimum horizontal range of rock type 4.

With the stationary and non-stationary mean, variance and covariance, non-stationary cokriging was performed with the FORTRAN program `kt3d_bound` (appendix). The kriging parameters used for the estimation include: a minimum and maximum of 4 samples, no octant search and an isotopic search radius of 650 meters.

```

Optimized Ranges
3
RT
Boundary_RT
Optimum_Range
2 3 326.957855
2 4 660.023438
2 5 465.359070
3 4 425.019592
3 5 335.151306
4 5 576.773804

```

Figure 7: Output file of opt_cov program.

In the first run a significant number of estimates were unusually high or low due to unusual kriging weights. These blocks were concentrated at the edges of the boundary zones. They represent a non-physical covariance model that must be reviewed as part of the future work. Two modifications to the kriging system were made to make the kriging matrix stable for the purpose of this application. A modification is required when the covariance of the estimate to data is higher than the covariance of the data to itself. This occurs because the estimate is inside a boundary zone and therefore has a non-stationary component that is added to the stationary covariance model, while the data is outside. To fix this, the diagonal term corresponding to the covariance of the sample to itself was set to 1.1 multiplied by the corresponding element in the estimate-to-data vector of the kriging system. When the estimation variance is calculated to be negative, all the diagonal terms of the data-to-data matrix were replaced by the maximum value of all elements in the kriging system. Additional to the modifications to the program a relatively small maximum number of samples were used to estimate a block. The origin of these non-physical results needs to be reviewed in the future. These relatively rare problems in kriging will become more important for the implementation of this technique in simulation where the correct estimate and estimation variance are essential for the reproduction of the conditioning data and its variability.

The correlation between the estimates and the reference or ‘true’ value is around 0.8 for each boundary zone (Figure 8). The stationary portions of rock type 2 and 5 show more conditional bias than rock types 3 and 4; the correlation between the estimate and reference ranges from 0.52 up to 0.8 (Figure 9). The mean of the reference in the stationary portions of each rock type is reproduced almost exactly by the kriging estimation (Table 2). The variance of the estimate is lower than the reference, as expected since kriging has a smoothing effect. The non-stationary behavior of the mean is also very well reproduced by the proposed non-stationary cokriging as shown in Figure 10. Although the variance of the estimates in the boundary zone is lower than the reference, the increasing trend toward the boundary is well reproduced (Figure 11).

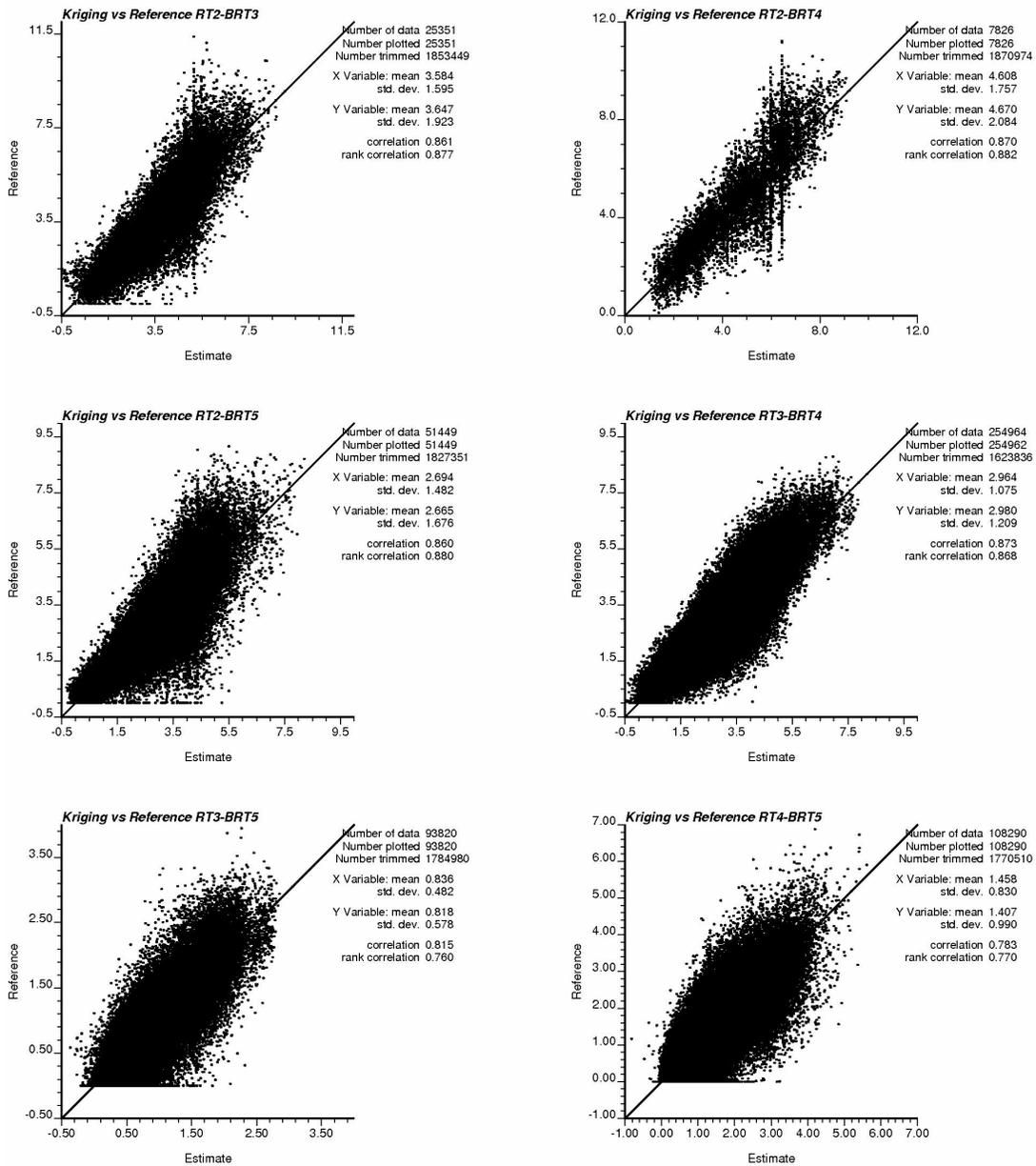


Figure 8: Scatter plot of the reference versus the estimate for each boundary zone.

Rock type (stationary region)	Reference		Kriging	
	Mean	Variance	Mean	Variance
2	3.13	1.03	3.14	0.56
3	0.92	0.52	0.91	0.39
4	1.66	0.71	1.66	0.57
5	0.28	0.21	0.27	0.14

Table 2: Comparison of the mean and variance of the each rock type (stationary regions) between the reference training image and the kriging results.

Validation of the model and parameters was done running `kt3d_bound` in the cross validation mode removing the entire drill hole to which the sample belongs. In this mode, the correlation between the estimate and the true value is 0.93. The results show that the resultant model is accurate and precise (Figure 12), the distribution of the error (estimate minus true) is symmetric and centered in zero and has a relatively small standard deviation.

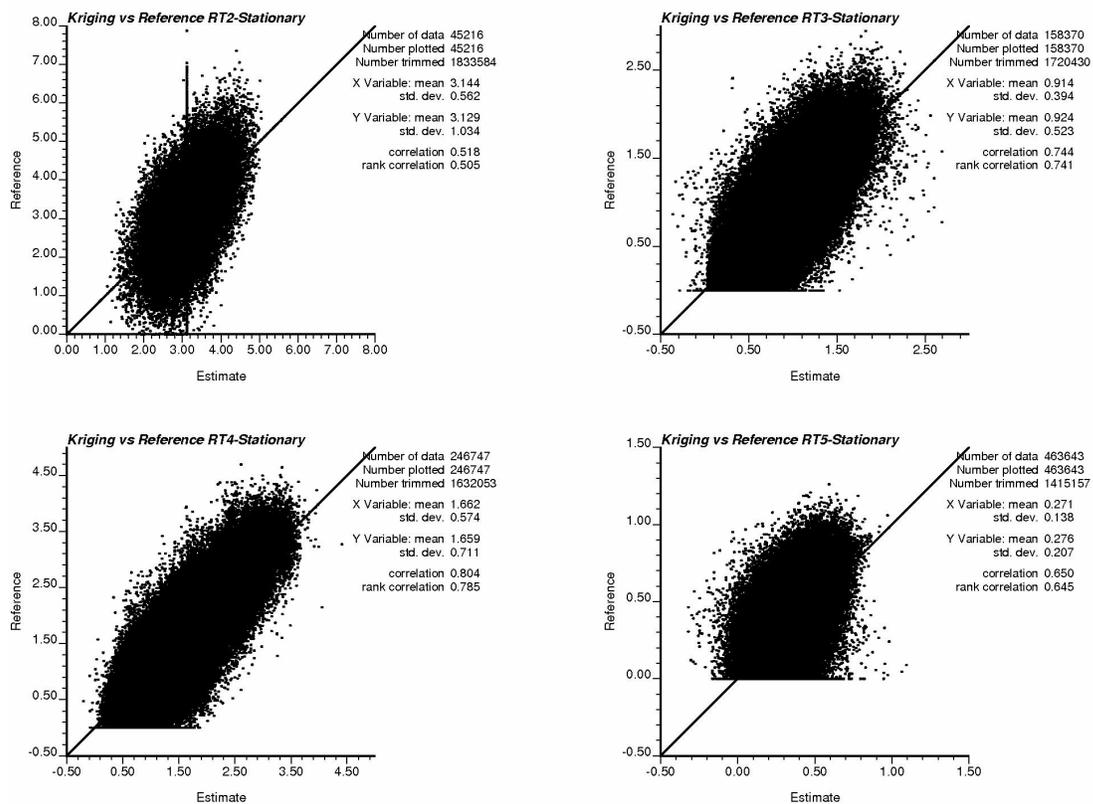


Figure 9: Scatter plot of the reference versus the estimate within the stationary portions of each rock type.

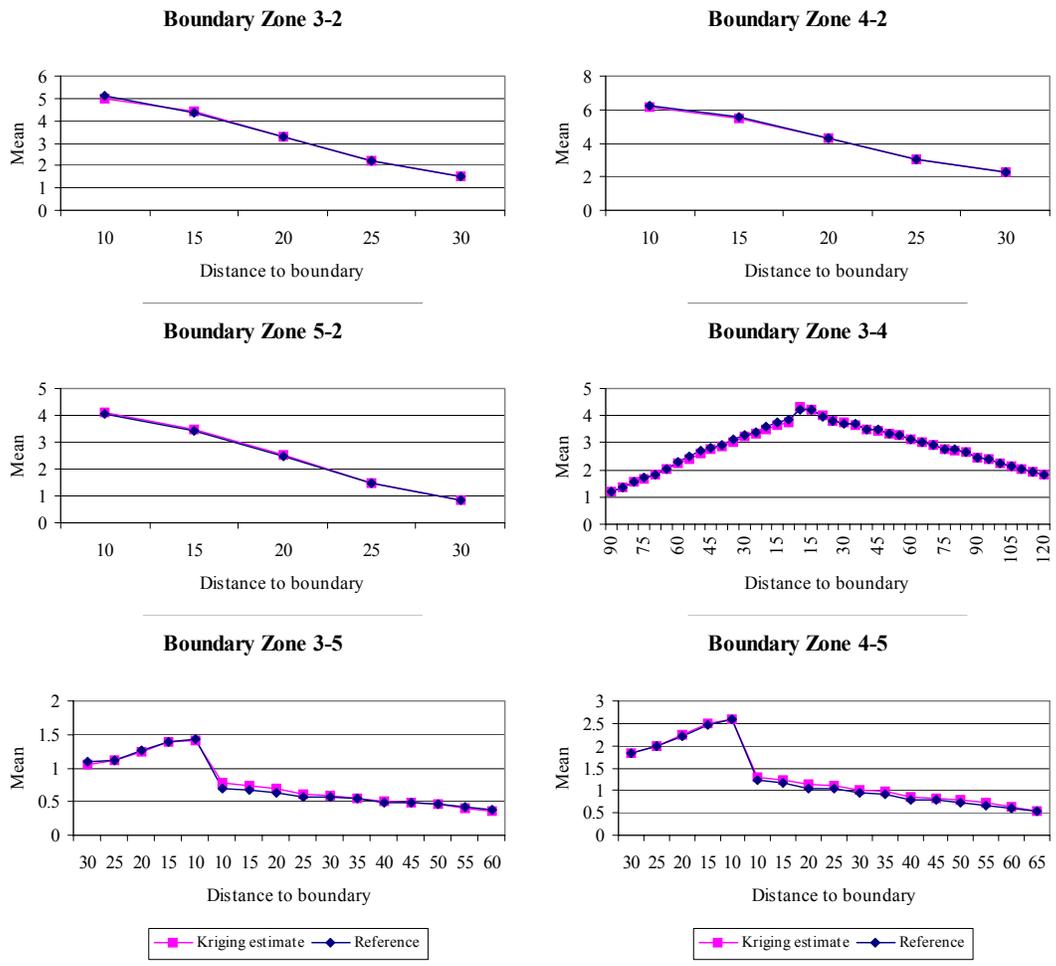


Figure 10: Mean at the non-stationary boundary zone. A 5 meters interval of the distance to the boundary was chosen to calculate the mean of the estimate value of all grid nodes.

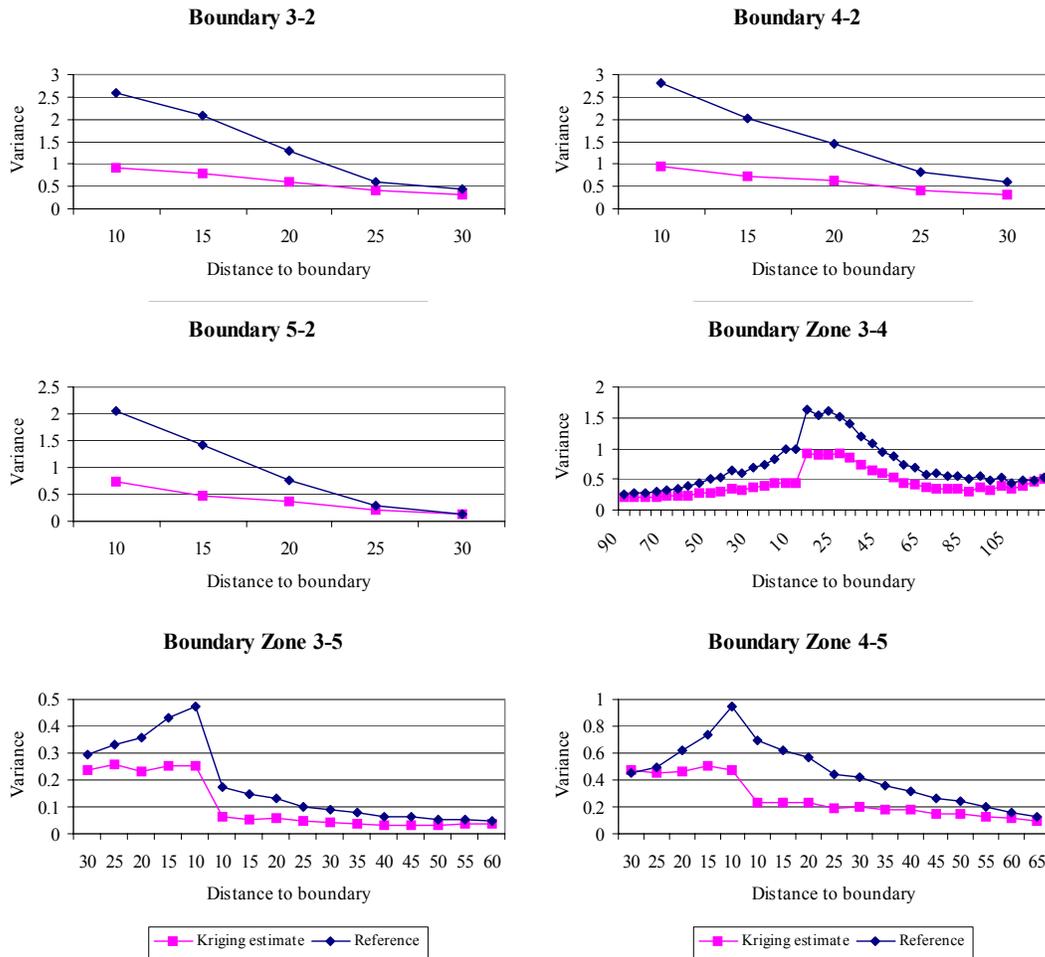


Figure 11: Variance at the non-stationary boundary zone. A 5 meters interval of the distance to the boundary was chosen to calculate the variance of the estimate value of all grid nodes.

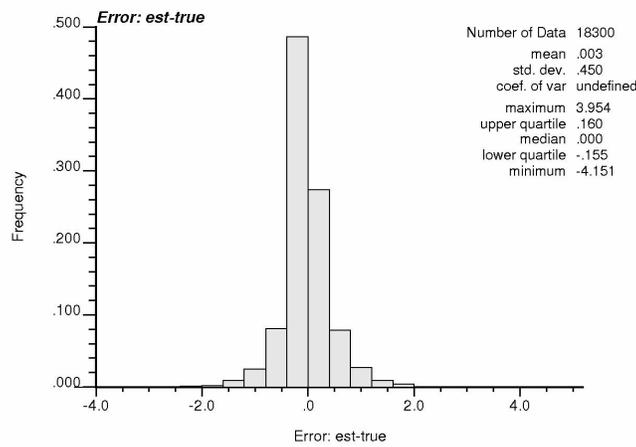
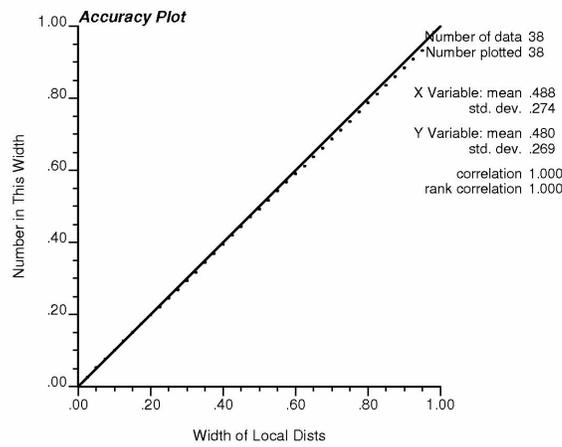
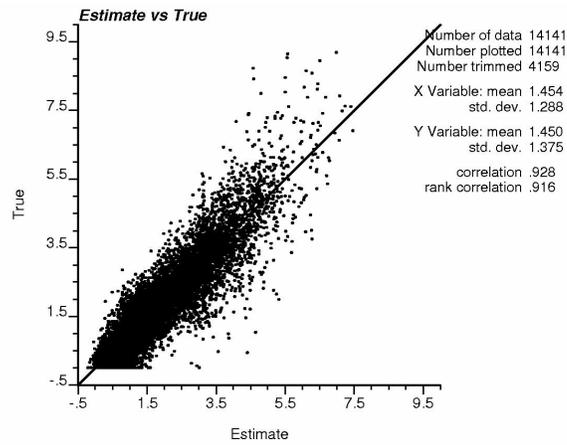


Figure 12: Cross validation results. Scatter plot of the estimate versus the true; the correlation is 0.92. The accuracy plot shows that the model is accurate and precise for the chosen parameters. The error (true-estimated) distribution is symmetric and centered in zero and with a relatively low standard deviation.

Conclusions and Future Work

Non-stationary cokriging in the presence of geological boundaries provides a theoretically robust methodology to handle non-stationary soft boundaries. With a large 3D geological model of a porphyry copper deposit the utility of this technique has been shown for a practical application.

The kriging estimates reproduce the non-stationary behavior of the conditioning data at the geological contacts, and it also reproduces the stationary means of each rock type in the model. A decrease in the global variance is due to the smoothing effect of kriging. Cross validation results show that the result works much better than assuming a hard boundary. The exceptions are some blocks at the edges of the boundary zones where non-physical results are obtained due to unusual kriging weights. They seem to originate from the linear model of coregionalization; this must be explored in further detail.

Acknowledgements

We would like to acknowledge the industry sponsors of the Centre for Computational Geostatistics at the University of Alberta for supporting this research.

References

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- P. F. Larrondo and C. V. Deutsch. Methodology for Geostatistical Model of Gradational Geological Boundaries: Local Non-stationary LMC. In *Centre For Computational Geostatistics*, volume 6, Edmonton, AB, 2004.

Appendix: Algorithms Implementation Details

RT model and boundaries

The FORTRAN program `boundmod` labels every location with the corresponding rock type and relevant closest precedent boundary zone and its distance to the boundary. The input data is a grid-type file of the geological model with the same format as the one used in GSLIB (Deutsch and Journel, 1998), the number of rock types available in the model, the matrix with all maximum distances of influence between rock types, and the set of precedence rules (Larrondo and Deutsch, 2004). The output file is a grid file with the rock type (from input), the boundary zone identified by the rock type code of the precedent influencing rock type, and the distance to the boundary with the precedent rock type.

The program loops over all nodes of the geological model grid, and checks within a search window if different rock types exist within the surrounding nodes. The cell may be assigned with a neighbor rock type if a boundary zone is defined for the two codes and the precedence rules are met. Detailed structure of this and the other programs presented below program in Larrondo (2004).

The matrix of maximum distances of influence is entered as lines with the corresponding maximum distance of influence of a specific rock type over all others. It has as many lines as different rock types in the model. If two rock types are not in contact with each other or a 'hard' boundary between them is assumed, the maximum distance of influence is set to zero, as in $dmax_{23}=0.0$.

The specification of the precedence rules is made through the total number of rules and a set of pairs of rock type codes. The order of the pairs is from oldest to youngest, or from less to more precedent, and is given by: "rock type code – boundary code". Leaving the number of precedence rules to zero is equivalent to choose a neutral arrangement where the precedent rock type is chosen as the one with the minimum distance to a boundary.

Mean optimization

The FORTRAN program `opt_mean` calculates the stationary component of the mean for each rock type code and the optimum intercept a_{kp} for each pair: rock type – boundary zone. The parameter a_{kp} , is the intercept at a distance from the boundary of zero (Larrondo and Deutsch, 2004).

The program first calculates the stationary means for each rock type, calculating the average over all samples that are within the internal stationary portion, that is, outside any of the boundary zones within the rock type. Then, a boundary zone and parameter a_{kp} is chosen randomly and perturbed within 20% of its original value. If the objective function decreases the proposed change to a_{kp} is accepted; if not, the original value of the parameter a_{kp} from the previous iteration is restored.

The input data is a file with the samples tagged with rock type, boundary zone and distance to the boundary; the number of rock types and the matrix of maximum distances of influence.

Variance optimization

The variance optimization is performed similarly to the mean, by a FORTRAN program called `opt_var`. The inputs are the same as for the `opt_mean` program plus the output file from the optimization of the means with the stationary means and the intercepts for the non-stationary means. The program uses the analytical expression for the non-stationary means for the different boundary zones and the corresponding stationary means to find the stationary variances and the optimum intercept b_{kp} that define the linear expression of the non-stationary variance for each boundary zone.

The optimization procedure has the same structure as the mean optimization program, but works with the residuals, calculated from the optimized mean. The optimization of the non-stationary variance is done through the optimization of b_{kp} .

Covariance optimization

The covariance optimization corresponds to find the optimum range of the pseudo stationary model that best fits the experimental spatial correlation of all pairs within a boundary zone. The FORTRAN program that finds the optimum range is called `opt_cov`.

The inputs for this optimization are: (1) a data set tagged with the rock type, boundary zone and distance to the boundary, (2) the stationary components of mean and variance for each rock type plus the intercepts that define the non-stationary components of this statistics for each boundary zone, (3) the number of rock types and matrix of maximum distances of influences, (4) the direct variogram models of the stationary portions of each rock type, and (5) a proposed model (shape, relative nugget effect and anisotropies) for the cross variograms given by the user based on the residuals spatial correlation at the boundary. The anisotropy is specified and fixed through the ratio between the initial ranges input by the modeler.

The program loops over all possible boundary zones. Since the modeler fixes the shape, nugget effect and anisotropies, the optimization of the non-stationary covariance model is done through the optimization of the range. The range is iteratively perturbed for each boundary zone until the objective function is minimized. The experimental covariance, calculated as the multiplication of the head and tail residual values, is compared against the stationary plus non-stationary covariances derived from the direct variograms models and the proposed cross variogram model to be optimized.

Non-stationary cokriging within boundaries

The FORTRAN program `kt3d_bound` allows to estimate unsampled locations in a geological model where non-stationary boundary zones have been identified, by using a covariance model that takes into account both the stationary and non-stationary components of the conditioning data. It also considers that the mean and variance have non-stationary components at locations within the boundary zones.

The inputs to this program include a data file and rock type model tagged by the corresponding rock type, boundary zone and distance to the boundary of each location, the statistical optimized models for the mean, variance and covariance and the kriging parameters. The output is a GSLIB grid-type file with the estimates and kriging variances for all locations within the grid definition of the model.

The program is a modified version of the GSLIB program `kt3d` (Deutsch and Journel, 1998) that performs simple kriging and/or cross validation using samples from the geological domain of the location to estimate and from adjacent domains. The data-to-data covariance matrix and the data-to-estimate covariance vector are filled according to the spatial configuration of the samples and location being estimated in relation with their location outside or inside a boundary zone and their distances to the boundary. A detailed scheme of how the covariance matrices and vectors are filled in this type of kriging is presented in Larrondo (2004). Once the kriging system is solved, using the `kt501` routine of GSLIB, the mean: stationary plus non-stationary component, of each sample and the location to be estimated are calculated in order to compute the solution. To calculate the kriging variance, the block covariance is calculated for each location to be estimated, as the stationary covariance of the corresponding rock type plus the non-stationary covariance if the block is within a boundary zone.